

Ex.5 证明. 考虑  $x_1\beta_1 + x_2\beta_2 + x_3\beta_3 = 0$ . 因为

$$\begin{aligned}\beta_1 &= 3\alpha_1 - \alpha_2 + \alpha_3, \\ \beta_2 &= 2\alpha_1 + 3\alpha_2 - \alpha_3, \\ \beta_3 &= 5\alpha_1 + 6\alpha_2 + 2\alpha_3.\end{aligned}$$

所以,

$$x_1(3\alpha_1 - \alpha_2 + \alpha_3) + x_2(2\alpha_1 + 3\alpha_2 - \alpha_3) + x_3(5\alpha_1 + 6\alpha_2 + 2\alpha_3) = 0.$$

即

$$(3x_1 + 2x_2 + 5x_3)\alpha_1 + (-x_1 + 3x_2 + 6x_3)\alpha_2 + (x_1 - x_2 + 2x_3)\alpha_3 = 0.$$

因为向量组  $\alpha_1, \alpha_2, \alpha_3$  线性无关, 所以,

$$\begin{cases} 3x_1 + 2x_2 + 5x_3 &= 0, \\ -x_1 + 3x_2 + 6x_3 &= 0, \\ x_1 - x_2 + 2x_3 &= 0. \end{cases} \quad (1)$$

系数矩阵的行列式

$$\begin{vmatrix} 3 & 2 & 5 \\ -1 & 3 & 6 \\ 1 & -1 & 2 \end{vmatrix} = 18 + 5 + 12 - 15 + 18 + 4 = 42 \neq 0.$$

所以齐次线性方程组(1) 只有零解, 所以, 向量组  $\beta_1, \beta_2, \beta_3$  线性无关.